A QUESTION CONCERNING THE LONGITUDINAL IMPACT OF A BODY ONTO A ROD

S. A. Bublik and I. G. Kadomtsev

UDC 539.3

In a quasi-static formulation we consider the longitudinal elastic-plastic impact of a body onto a semi-infinite rod. An elastic solution of this problem is well known, namely, that proposed in [1] by Sears; however, his solution is confirmed by experimental data only for small impact speeds. With an increase of impact speed plastic deformation appears in the region of contact which has a noticeable effect on the basic impact parameters: the contact interaction force P, the local deformation α, and the time of contact.

In a precise formation the problem concerning nonelastic impact leads to a dynamic elastic-plastic problem which, by virtue of its complexity, may be solved either numerically or approximately. Assuming that the speed of impact is much less than the speed of sound in the bodies, we can neglect inertias of local deformation and solve the problem in a quasi-static formulation, i.e., we can assume that in the dynamic problem the function $\alpha(P)$ stays the same as in the static problem. It is assumed in the present paper that total displacements of the rod can be considered as elastic, and that deformations in the region of contact of the body and the rod can be treated as elastic-plastic. We employ an earlier-developed model for the local deformation $\alpha(P)$ of axially symmetric elastic-plastic bodies which differs from previous models in that in it the outflow of material from the contact zone is taken into account and plastic deformations are accounted for from the instant that the mean stresses in the contact zone reach the Brinell stage [2].

We locate the coordinate origin at the point of initial contact of the body and the rod. The equation for longitudinal oscillations of the rod has the form

$$\partial^2 u_1/\partial^2 t^2 = a_1 \partial^2 u_1/\partial x^2,\ a_1 = (E_1/\rho_1)^{1/2}. \tag{1}$$

Here u_1 is the longitudinal displacement of points of the rod; E_1 is the Young's modulus; ρ_1 is the density of the material of the rod.

A displacement of the body is described by the equation

$$m\partial^2 u_2/\partial t^2 = -P(t),$$

where u_2 is the displacement of the body; m is its mass; P(t) is the force of interaction of the body and the rod at contact. Initial conditions are the following:

$$u_1(x, 0) = 0,\ u_2(x, 0) = 0,\ \partial u_1(x, 0)/\partial t = 0,\ \partial u_2(x, 0)/\partial t = v_0$$

(v_0 is the initial speed of impact). Local deformation has the form

$$\alpha = u_2(0, t) - u_1(0, t). \tag{2}$$

At the end of the rod we have the condition

$$E_1 F_1 \partial u_1(0, t)/\partial x = -P(t) \tag{3}$$

(F_1 is the area of a rod cross section).

We apply a Laplace transform to Eqs. (1) and (3):

$$\partial^2 U_1/\partial x^2 = x^2 U_1/a_1^2; \tag{4}$$

$$sU_1(0, s) = a_1 Q/(E_1 F_1), \tag{5}$$

$$U_1 = s\int_0^\infty u_1(x, t)\exp(-st)\,dt,\ Q = s\int_0^\infty P(t)\exp(-st)\,dt$$

(s is the Laplace transform parameter). It follows from relations (4) and (5) and the condition $U_1(\infty, s) = 0$ that

Rostov-on-Don. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 123-125, January-February, 1992. Original article submitted April 6, 1990; revision submitted October 12, 1990.

0021-8944/92/3301-0109$12.50

$$u_1 = C_1 \exp(-sx/a_1),\ C_1 = Qa_1(E_1F_1s)^{-1}. \qquad (6)$$

Transforming relation (6), we obtain

$$\partial u_1(0, t)/\partial t = a_1P(t)/(E_1F_1).$$

Thus the initial problem is reduced to the Cauchy problem

$$m\partial^2 u_2(0, t)/\partial t^2 = -P(t),\ \partial u_1(0, t)/\partial t = a_1P(t)/(E_1F_1),$$
$$u_1(0, 0) = 0,\ u_2(0, 0) = 0,\ \partial u_1(0, 0)/\partial t = 0,\ \partial u_2(0, 0)/\partial t = v_0. \qquad (7)$$

Here the condition (2) must be satisfied. In the Sears theory the function $\alpha(P)$ is elastic, i.e.,

$$\alpha = hP^{2/3},\ h = R^{-1/3}(3/(4E))^{2/3},\ R^{-1} = R_1^{-1} + R_2^{-1},$$
$$E = E_1E_2/[(1-\nu_1^2)E_2 + (1-\nu_2^2)E_1]. \qquad (8)$$

Here R_2 and R_1 are the radii of curvature of the body and of the end of the rod at their point of contact; ν_1 and ν_2 are Poisson coefficients for the material of the rod and body.

In proceeding we use the model from [2] for the local deformation, writing it in the dimensionless form

$$\frac{\alpha}{\alpha_0} = \begin{cases} P_*^{2/3}, & P_* < 1 \\ [(1+\beta)P_*^{1/2} + (1-\beta)P_*]/2, & dP/dt > 0 \\ P_*^{2/3}P_{\max}/P_1 + (1-\beta)[P_{\max}/P_1 - P_{\max}^{1/2}P_1^{-1/2}]/2, & dP/dt < 0 \end{cases} \quad P_* > 1, \qquad (9)$$

where $P_* = P/P^0$; P^0 and α^0 are the contact force and local deformation, starting from which local deformations are taken into account; P_{max} is the largest force attained during the penetration stage; $P^0 = \varkappa R\alpha^0$; $\alpha^0 = R[3\varkappa/(4E)]^2$; $\varkappa = \pi k\gamma$; $k = \sigma^0/2$; $\gamma = 5.7$ in the absence of friction between the bodies; β characterizes flowing out of the material from under the stamp during the penetration process (if outflow is not taken into account, then $\beta = 0$; in the absence of friction, $\beta = 0.33$ for a parabolic stamp); k is the plastic constant.

For developed plastic deformations we can obtain an approximate solution of the initial problem in closed form. In relation (9), when $dP/dt > 0$ the linear term in P will be the dominant term for plastic deformations, and we can then take

$$\alpha = bP,\ b = \lambda(2R\pi k\gamma)^{-1},\ \lambda = 1 - \beta. \qquad (10)$$

In this case relation (7) becomes

$$\ddot{\alpha} + a_1\dot{\alpha}(E_1F_1)^{-1} + \alpha(bm)^{-1} = 0,\ \alpha(0) = 0,\ \dot{\alpha}(0) = v_0,$$

whence it follows that

$$\alpha = v_0r^{-1}\exp(-dt)\sin(rt),\ d = a_1(2bE_1F_1)^{-1},$$
$$r = [(bm)^{-1} - d^2]^{1/2}. \qquad (11)$$

The penetration time T is determined from the condition $\dot{\alpha}(T) = 0$; hence, $\tan(rT) = 2[(d^2 \times bm)^{-1} - 1]^{1/2}$. Since d^2bm is a small quantity, we can assume that $rT = \pi/2$, from which it follows that

$$T = \pi[\lambda m(8R\pi k\gamma)^{-1}]^{1/2},\ \alpha_{\max} = v_0r^{-1}\exp(-dT)\sin(rT).$$

Here α_{max} is the maximum local deformation corresponding to the largest penetration force P_{max}. Since $\sin(rT) \to 1$, it follows from relations (11) that α_{max} has the form

$$\alpha_{\max} = v_0r^{-1}\exp[-\pi(4F_1)^{-1}(mR\pi\sigma^0\gamma)^{1/2}(\lambda E_1\rho_1)^{-1/2}].$$

From relations (10) we have $P_{max} = \alpha_{max}/b$.

Finally, we find that the basic impact parameters in this case are given by the expressions

$$\alpha_{\max} = v_0(m\lambda)^{1/2}(R\pi\sigma^0\gamma)^{-1/2},\ P_{\max} = v_0\lambda^{-1/2}(mR\pi\sigma^0\gamma)^{1/2},$$
$$T = \pi[\lambda m(8R\pi k\gamma)^{-1}]^{1/2}. \qquad (12)$$

Problem (7), with function $\alpha(P)$ in the form (8), (9), and also with $\alpha(P)$ in the form given in [3], was solved numerically by the Runge–Kutta method. Here we determined the basic impact parameters. All the initial data for our calculations were taken to be those used in the

TABLE 1

l, m	m, kg	R, m	v_0, m/sec	P_1	P_2	P_3	P_4	P_5	T_1	T_2	T_3	T_4	T_5
				kN					$c\cdot 10^{-3}$				
0,9	20	0,065	3,65	335	399	222	356	324	0,354	0,310	0,518	0,322	0,348
0,6	13,1	0,063	1,73	120	128	80	134	121	0,492	0,310	0,459	0,265	0,298
0,6	13,1	0,063	3,84	286	331	188	298	273	0,274	0,260	0,427	0,265	0,287
0,6	13,1	0,063	5,20	397	474	260	404	372	0,240	0,249	0,417	0,265	0,280
0,3	4,86	0,063	4,20	193	207	123	199	182	0,197	0,175	0,270	0,161	0,175
0,3	4,86	0,063	5,56	252	289	266	263	243	0,165	0,165	0,260	0,161	0,170
0,3	4,86	0,063	6,50	300	349	196	308	286	0,149	0,160	0,258	0,161	0,170

experiments reported in [4], and the results of our calculations were compared with the experimental results. Steel rods of length ℓ and mass m fell with initial speed v_0 onto a base of duraluminum D1-T. The ends of the rods were curved (with radius of curvature R). The basic impact characteristics are shown in Table 1. Here P_1, P_2, P_3, P_4, and P_5 are the maximum values of the contact force as determined, experimentally in [4], by the Sears model in [1], by Kil'chevskii's elastic-plastic model in [3], and by the rigid-plastic and elastic-plastic local deformation models, respectively; T_1, T_2, T_3, T_4, and T_5 are the times of impact. Comparing results, we see that the Sears theory, based on an elastic model due to Hertz, gives a value for P_{max} greater, on the average by 20-30%, in comparison with the experimental value, and it gives a lower value for T. The theory, based on Kil'chevskii's elastic-plastic model [3], yields a value for P_{max} lower by 30-40% and a larger value for T. The theory proposed here yields results which differ from the experimental results by 2-6%. In the case of the rigid-plastic model, a particular case of the elastic-plastic model, the basic impact parameters are given in the explicit form (12) and results of calculations differ from experiment by 2-12%.

LITERATURE CITED

1. W. Goldsmith, Impact, Theory and Physical Properties of Colliding Bodies [Russian translation], Stroiizdat, Moscow (1965).
2. V. M. Aleksandrov, I. G. Kadomtsev, and L. B. Tsaryuk, "Axially symmetric contact problems for elastic-plastic bodies," Frict. Wear, 5, No. 1 (1984).
3. N. A. Kil'chevskii, Dynamic Contact Compression of Solid Bodies Impact [in Russian], Naukova Dumka, Kiev (1976).
4. G. S. Batuev, Yu. V. Golubkov, A. K. Efremov, and A. A. Fedosov, Engineering Methods for the Study of Impact Processes [in Russian], Mashinostroenie, Moscow (1977).

COMPACT REPRESENTATION OF THE FUNDAMENTAL SOLUTION OF THE INTERNAL LAMB PROBLEM ON A FREE SURFACE

A. S. Tyapin and A. I. Startsev

UDC 534.16+550.344.5

The now-classical expression derived [1] for the Green's function of the problem of ground surface displacements induced by an explosion in a homogeneous containment medium is written in the form of a three-term sum. One of the terms is the Boussinesq solution for a half-space, which assumes that the disturbance propagates instantaneously [2]; another term contains typical Rayleigh components, and the third represents certain real integrals. This representation of the Green's function is convenient from the standpoint of the physical treatment of the propagation of seismic waves in a medium and affords a rapid and efficient means of calculating the displacements far from the wave front. However, it is nonoptimal for calculating the displacements near the wave front at a large distance from the detonation epicenter, where the indicated terms strongly suppress one another in the vicinity of the front.

In the present article we describe an attempt to surmount this difficulty by deriving a more compact representation of the Green's function in question without incurring such a

Arzamas. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 126-132, January-February, 1992. Original article submitted July 23, 1990.